

General issues in small- x and diffractive physics^{1,2}

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Abstract. A selected review of topics at the border of hard and soft physics is given. Particular emphasis is placed on diffraction dissociation at Fermilab and HERA. Recently, significant differences between diffraction dissociation at HERA and at Fermilab have become apparent. This may suggest that one already is reaching nonlinear (unitarity) effects which are extending from the soft physics region into the semihard regime of QCD.

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1 Introduction

The focus in this paper is on the regime of hardness near the borderline between hard and soft high energy collisions with a special emphasis on searching for nonlinear QCD effects. This is an opportune time for such a discussion as there is now a significant body of complementary data from deep inelastic scattering and from hadron-hadron collisions. Perhaps the major object of this review is to compare and contrast hadronic and deep inelastic collisions, especially diffraction dissociation where there are major differences between the hadronic and virtual photon initiated processes.

Section 2 is devoted to a brief review of some soft physics results on total cross sections and diffraction dissociation. Despite the fact that total cross sections grow [1] as $(s/s_0)^\epsilon$, with $\epsilon \approx 0.1$, through the highest Fermilab energies it is argued that there already is strong evidence of unitarity corrections being important from the ISR to Fermilab energy regimes. In particular, we suggest the lack of growth of the single diffraction dissociation cross section [2-5] as due to the blackness of central proton-proton and proton-antiproton collisions.

In contrast to a very weak growth of the single diffraction dissociation cross section in hadronic collisions the energy dependence of virtual photon diffraction dissociation appears to be significantly stronger [6-7] than that expected from soft physics. In Sect. 3, we argue that this may be due to blackness for the soft components of the virtual photon's wavefunction and a subsequent dominance of the process by semihard components as suggested recently by Gotsman, Levin and Maor [8]. If this is indeed

the case it means that for the first time one has evidence of unitarity (nonlinear) effects extending into the semihard regime of QCD.

In Sect. 4, we remark that the new $D\bar{O}$ data [9,10] on rapidity gaps between jets showing that the gap fraction of events decreases with energy may be due to the same physics which slows the growth of the single diffraction cross section. If this decrease is indeed due to an energy dependent (decreasing) survival probability a similar behavior would be expected for comparable photoproduction data involving resolved photons, but such a decrease would not be seen in deep inelastic scattering.

In Sect. 5, we review BFKL searches performed at H1, ZEUS and $D\bar{O}$ [9,11-13]. Each analysis, using two-jet inclusive measurements at Fermilab and a forward jet measurement at HERA, finds some evidence for BFKL behavior through an energy dependence which seems stronger than expected from leading and next-to-leading order perturbation theory. However, definitive results have not yet been achieved.

In Sect. 6, progress in calculating the next-to-leading corrections in BFKL evolution is reviewed [14]. We may be near a rather complete understanding of these corrections. A preliminary estimate suggests a substantial reduction of the BFKL pomeron intercept.

Sect. 7 is devoted to a brief discussion of some topics involving nuclear reactions. Parametrizations of diffraction dissociation at HERA have been successfully used to describe nuclear shadowing in fixed target deep inelastic lepton-nucleus scattering [15]. J/ψ production in proton-nucleus and nucleus-nucleus scattering continues to be an important subject for research. New phenomenological success in describing all data except for Pb-Pb collisions by a simple absorption model [16,17], along with the suggestion that the Pb-Pb data may be qualitatively different [18] have made it even more important to connect J/ψ production and scattering more firmly with QCD.

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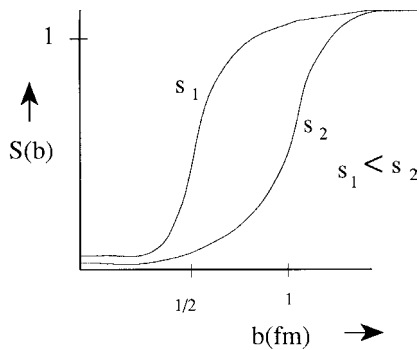


Fig. 1. The S-matrix as a function of impact parameter for two energies $s_1 < s_2$

2 Clearly soft

2.1 Total cross sections

Donnachie and Landshoff [1] have shown that all high energy total cross sections for hadron-hadron collisions can be written in the form

$$\sigma_{tot} = \sigma_0 (s/s_0)^\epsilon + \text{subleading terms} \quad (2.1)$$

where σ_0 depends on the particular hadrons initiating the collision and the subleading terms go to zero roughly like $(s/s_0)^{-1/2}$. s_0 is an arbitrary scale factor while ϵ appears to be universal and of size

$$\epsilon \approx 0.1. \quad (2.2)$$

$1 + \epsilon = \alpha_p$ is the intercept of the soft pomeron in Regge-language. HERA data shows that (2.1) is also true for real photon-proton collisions. Of course a growth in energy as fast as that indicated in (2.1) cannot persist at arbitrarily high energies because of limitations required by the Froissart bound which does not permit total hadronic cross sections to rise faster than $\ln^2 s/s_0$ at asymptotic energies. The fact that the behavior indicated in (2.1) persists up to the highest Fermilab energy region might seem to indicate that unitarity constraints, which are responsible for the Froissart bound, are not yet effective in the presently available energy region. However, as observed long ago [19], this is not the case. If one writes proton-proton total, inelastic and elastic cross sections in terms of the S-matrix at a given impact parameter of the collision, $S(b)$, as

$$\sigma_{in} = \int d^2b [1 - S^2(b)] \quad (2.3)$$

$$\sigma_{el} = \int d^2b [1 - S(b)]^2 \quad (2.4)$$

$$\sigma_{tot} = 2 \int d^2b [1 - S(b)], \quad (2.5)$$

then $S(b)$ depends on b roughly as indicated in Fig. 1. (We take $S(b)$ to be real for simplicity.)

For small values of impact parameter $S(b)$ is near zero for proton-proton collisions already in the ISR and Fermilab fixed target energy regime. For proton-antiproton

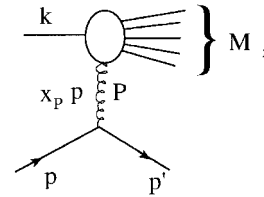


Fig. 2. Diffractive Excitation via soft pomeron exchange

collisions $S(b)$ is quite small for $b < 1 fm$ in the Fermilab energy regime. $S(b)$ near zero is a signal that unitarity corrections are large though they are not so easy to see in the total cross section because the radius of interaction is expanding and the growth of σ_{tot} is mainly coming from that expansion. Monte Carlo simulations [20] of the dipole formulation [21,22] of the Balitsky, Fadin, Kuraev, Lipatov (BFKL) [23,24] equation for the academic case of heavy onium-heavy onium scattering show a similar phenomenon. For rapidities less than about 15 the BFKL equation is reliable for the total onium-onium cross section, however, for small impact parameter collisions important unitarity corrections are visible for rapidities of 6 units.

2.2 Diffraction dissociation in hadron-hadron scattering

Single diffraction dissociation, illustrated in Fig. 2, is given by

$$x_P \frac{d\sigma_{SD}}{dx_P dt} = x^{2(1-\alpha_P(t))} f(M_x^2) \quad (2.6)$$

is terms of soft pomeron exchange. Integrating (2.6) over t and over $x_P \leq 0.05$, but excluding the proton state $M_x = M_p$, one gets a single diffractive cross section σ_{SD} . From the Regge formalism one expects σ_{SD} to grow with s as $s^{2\epsilon}$, but this is not seen in the data as illustrated in Fig. 3, which is a simplified version of the more complete plot in [5] where detailed data points are shown. A factor of 2 is included in Fig. 3 to account for single diffractive dissociation of either of the colliding protons (antiproton). At Fermilab collider energies there is a discrepancy of an order of magnitude between the Regge fit and the data.

It seems clear that this discrepancy and the slow growth of σ_{SD} with energy signal a breakdown of the Regge analysis when $\sqrt{s} \geq 20 GeV$. I think this breakdown can be expressed in various equivalent ways. (i) At low energies inelastic collisions induce, through unitarity, both elastic scattering and diffractive dissociation. However, as $S(b)$ goes to zero for small and moderate b at high energies, these black regions of impact parameter space only induce elastic scattering and not diffraction dissociation. Thus as one increases energy the elastic cross section grows rapidly while the diffraction dissociation cross section, coming from those regions in impact parameter space where $S(b)$ is neither too close to zero or too close to one, grows very slowly. (ii) In the Regge language one must include multiple pomeron exchange in addition to the single pomeron exchange which is valid at lower energy. This multiple

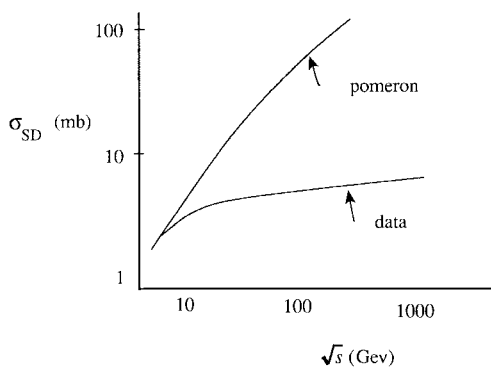


Fig. 3. A Schematic picture of the triple-pomeron prediction and the data for single diffractive cross section as a function of energy

pomeron exchange gives absorptive (virtual) corrections which slow the growth coming from single pomeron exchange. (iii) The “gap survival” probability [25,26] decreases with energy compensating the growth due to single pomeron exchange. Although gap survival probability is a concept usually used for hard collisions I think the same idea applies to single diffraction dissociation, at least in a heuristic way, in hadron-hadron collisions. It is likely that (i), (ii) and (iii) are just different ways of saying the same thing.

3 Deep inelastic lepton-proton scattering analogs of the soft physics results

3.1 An “elastic” scattering amplitude

Recently, there has been an interesting suggestion as to how to test unitarity limits in deep inelastic scattering [27]. Of course there is no Froissart bound for virtual photon-proton scattering, nevertheless, we have become used to viewing the small-x structure function in terms of a high energy quark-antiquark pair (possibly accompanied by gluons) impinging on the target proton. Although the quark-antiquark pair is not on-shell the time evolution of the pair as it passes through the nucleon should be constrained by unitarity in much the same way that a quark-antiquark pair coming from, say, a pion state would be.

To be more specific, view deep inelastic scattering in the rest system of the proton and in the aligned jet (naive parton) model [28,29]. At small x the virtual photon, $\gamma^*(q)$, breaks up into a quark and antiquark pair long before reaching the proton. The relative transverse momentum of the quark and antiquark is small, of hadronic size $\mu \approx 350 \text{ MeV}$, while the longitudinal momenta are q_z and $q_z \cdot \frac{\mu^2}{Q^2}$ respectively. Because the relative transverse momentum is small the transverse coordinate separation of the quark and antiquark can be expected to be on the order of a fermi, and the resulting cross section with the proton should be of hadronic size. The smallness of the overall cross section comes from the small probability, of or-

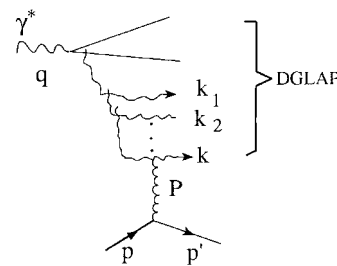


Fig. 4. Diffractive excitation of the virtual photon

der μ^2/Q^2 , to find such an aligned jet configuration in the wavefunction of γ^* . (More probable configurations in the γ^* have smaller interaction probabilities. While the aligned jet model cannot be expected to be a precise model of deeply inelastic scattering it should reasonably characterize a significant portion of deep inelastic events.)

The inelastic reaction of this, longitudinally asymmetric, quark-antiquark pair with the proton should produce a shadow quark-antiquark pair in the final state. If the center of the proton is relatively black to the incoming quark-antiquark pair the shadow may be rather strong, as in the hadronic case discussed above, and unitarity limits may already be reached at present energies. The outgoing quark-antiquark pair should show up as a diffractively produced state, of mass $M_x \approx Q$, following the direction of the γ^* . Assuming that the scattering amplitude of the quark-antiquark pair with the proton is imaginary one may reconstruct this amplitude, dropping an i , as

$$F(x, b) = \int d^2 p e^{i \mathbf{p} \cdot \mathbf{b}} \sqrt{\frac{d\sigma_{SD}}{d^2 p}} \quad (3.1)$$

with b the impact parameter of the collision and p the momentum transfer to the recoil proton. $\frac{d\sigma_{SD}}{d^2 p}$ is the single diffractive cross section for $M_x \approx Q$.

In the present circumstance we do not have good control of the magnitude of F near $b = 0$. However, if the proton is black for central collisions one can expect $F(x, 0)$ to show little x -dependence. (Here x plays the role that s does for the hadronic collisions discussed above.) The authors of [27] suggest looking at the b -dependence of

$$\Delta_{eff} = \frac{d \ln F(x, b)}{d \ln 1/x}. \quad (3.2)$$

Unitarity constraints can be expected to show up as smaller values of Δ_{eff} near $b = 0$. More quantitatively, unitarity limits at $b = 0$ would give

$$\Delta_{eff}(b = 0) < 2(\alpha_P - 1). \quad (3.3)$$

This is a clever idea, and it will be interesting to see what the data give.

3.2 Large mass γ^* diffractive dissociation

The traditional picture of large mass diffraction dissociation at small values of x is shown in Fig. 4, where Dokshitzer, Gribov, Lipatov, Altarelli, Parisi (DGLAP)[30-32]

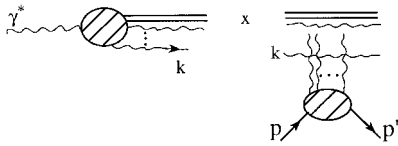


Fig. 5. Diffractive excitation of the virtual photon as a two-step process

evolution takes one from the hard scale Q to the soft scale μ where a soft diffractive scattering, represented by soft pomeron exchange, occurs. In Fig. 4, one assumes the DGLAP ordering

$$\mu^2 \approx k^2 \ll \dots \ll k_2^2 \ll k_1^2 \ll Q^2. \quad (3.4)$$

However, this is a subtle process and it is worthwhile looking carefully at the argumentation that leads to the size of k^2 at the lower end of the DGLAP evolution [8, 33-35]. It is convenient to view that evolution proceeding from the hard scale Q toward softer scales, a direction opposite to that which is usually taken. In Fig. 5, we illustrate the process in two steps: The left-hand part of the figure shows the virtual photon wavefunction in terms of its quark and gluon components. As in Fig. 4, k is supposed to be the softest gluon and $\Delta x_\perp = 2/k_\perp$ gives the transverse size of the γ^* state. The right-hand part of the figure gives the diffractive scattering part of the process proceeding by gluon exchange from the proton interacting with the octet dipole consisting of the gluon k and the remainder of the γ^* state. Schematically, one may write

$$d\sigma_{SD} = \text{flux } dP_r(k_\perp) [1 - S(\Delta x_\perp = 2/k_\perp, \underline{b}, Y = \ell n 1/x)]^2 d^2 b dx_P \quad (3.5)$$

where

$$dP_r(k_\perp) = \frac{dk_\perp^2}{Q^2} \quad (3.6)$$

is the probability that the lowest transverse momentum gluon have momentum k_\perp . (3.6) shows that gluons with small k_\perp have a small probability in the γ^* wavefunction, analogous to what we found earlier for low momentum quarks in the aligned jet model. $1 - S$ represents the amplitude for a gluon having k_\perp , along with the remainder of the γ^* wavefunction, to scatter elastically on the proton. b is the impact parameter of the overall collision while $Y = \ell n 1/x$ is the rapidity of the softest gluon with respect to the proton. In *lowest order*, two-gluon exchange,

$$1 - S \propto \frac{x_P G(x_P, k_\perp^2)}{k_\perp^2} \quad (3.7)$$

when k_\perp is large and where an integration has been performed over impact parameter, \underline{b} . Using (3.6) and (3.7) in (3.5) one sees, dimensionally, that k_\perp^2 cannot be large and this is the logic that has led theorists to take $k_\perp \approx \mu$ and use soft pomeron exchange for the scattering amplitude, $1 - S$.

However, if S is near zero for $k_\perp = \mu$ and for $\underline{b} = 0$, and this is not unreasonable since the S matrix is near

zero for small impact parameter hadron-hadron collisions, then it is apparent from (3.5) and (3.6) that values of k_\perp significantly larger than μ will be important. Indeed, the values of k_\perp that will dominate large mass single diffractive production are those values where S is near, but not too close to, one. This is the case since the probability in the γ^* wavefunction is located in large k_\perp values. The situation here is quite different than for hadron-hadron collisions. In hadron-hadron collisions the wavefunction of the incoming hadron is, except for a very small part, in the soft physics region. If the S -matrix is near zero for central collisions then the inelastic reaction will feed into elastic scattering as a shadow. In deep inelastic scattering at small- x when S becomes black there will certainly be a similar phenomenon which occurs, and which has been described in Sect. 3.1, but, in addition, blackness in the small k_\perp region will allow higher values of k_\perp to become effective thus making the process semihard.

If central impact parameter collisions of γ^* -proton collisions are indeed black for $k_\perp \approx \mu$ then we would expect the x -dependence of the single diffractive cross section, $x_P \frac{d\sigma}{dx_P}$, to vary more strongly with x than suggested by the soft pomeron. If one writes

$$x_P \frac{d\sigma}{dx_P} \propto x^{-n} \quad (3.8)$$

then both ZEUS [6] and H1 [7] now suggest that $n \approx 0.4$ rather than the $n = 2(\alpha_P - 1) \approx 0.2$ predicted by the soft pomeron. If the ZEUS and H1 measurements hold up, and n really is near 0.4 in the small β region, then I think it becomes clear that semihard physics is dominating the physics of large mass diffraction dissociation. In that case it is interesting to reexamine the ‘‘elastic’’ scattering analyses we described in Sect. 3.1 to see if the proposed procedure to measure blackness is also destroyed by the dominance of gluons and quarks at higher k_\perp -values. Finally, it should be pointed out that there are already rather detailed calculations of this phenomenon, at least for $q\bar{q}$ and $q\bar{q}g$ components of the γ^* wavefunction, which arrived at a value $n \approx 0.5$, not too far from experiment [8]. Have we, for the first time, actually seen the long sought after evidence for nonlinearity (unitarity limits) in the semihard region of deep inelastic scattering?

Before leaving this section, it may be useful to again contrast hadron-hadron scattering with γ^* -proton scattering. In the purely hadronic case the energy dependence of the single diffraction cross section is much weaker than that predicted by the soft pomeron. We have interpreted this as due to blackness in central proton-proton collisions which enhances the elastic cross section but suppresses diffractive excitation. In γ^* -proton scattering, on the other hand, the energy dependence (x -dependence) is much stronger than that predicted by the soft pomeron. We have interpreted that also as due to blackness of the soft components of the γ^* now leading to an enhanced role for the harder components of the γ^* -wavefunction and a resulting stronger energy dependence of the cross section.

4 Rapidity gaps between jets at Fermilab and HERA

Suppose one measures two jets having comparable but opposite transverse momentum along with the requirement that there be a rapidity gap between two jets. One might hope that this would be a good process to measure the hard (BFKL) pomeron as illustrated in Fig. 6 [36]. There are, however, at least two worries with using this process to measure the hard pomeron. (i) The pomeron contribution to the hard quark-antiquark scattering is [37].

$$\frac{d\sigma}{dt} = (\alpha C_F)^4 \frac{\pi^3 \exp[2(\alpha_P - 1)\Delta Y]}{4t^2 [\frac{7}{2}\alpha N_c \zeta(3)\Delta Y]^3} \quad (4.1)$$

with ΔY the rapidity between the two jets. Here $\alpha_P - 1 = \frac{4\alpha N_c}{\pi} \ln 2$ is the BFKL pomeron intercept. The presence of the factor $(\Delta Y)^3$ in the denominator in (4.1) strongly reduces the effective growth of the cross section with ΔY making the emergence of the hard pomeron more difficult at moderate values of ΔY . (ii) Perhaps more serious yet is the fact that the cross section for producing two jets with a gap between them depends on the absence of a soft interaction between the spectator parts of the proton and antiproton, the so-called gap survival probability [36]. This lack of factorization makes it difficult to make a precise comparison between theory and experiment.

There is new data from DØ [9, 10] and an interesting new analysis comparing the 1800 GeV data with that at 630 GeV. If f_{gap} is the fraction of all two-jet events (separated by a given rapidity) with a gap between them then DØ finds that

$$\frac{f_{gap}(630)}{f_{gap}(1800)} = 2.6 \pm 0.6_{stat}. \quad (4.2)$$

for $\Delta Y \geq 3.8$. Thus the gap fraction *decreases* with increasing energy. While this number cannot be directly compared to BFKL dynamics because ΔY has been taken to be the same at the two energies, while a BFKL test should have $\Delta Y(1800) - \Delta Y(630) = \ln \frac{1800}{630}$, it does suggest that the survival probability has a rather strong energy dependence making BFKL tests more difficult in rapidity gap events. It will be interesting to see whether models of the gap survival probability can easily accommodate the energy dependence in (4.2) [38].

At Fermilab the gap fraction is typically 0.01 while at HERA more like 0.07. The gap survival probability is

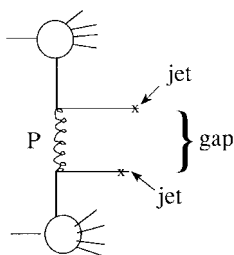


Fig. 6. Two-jet production with a gap between them

much larger at HERA as is natural for a point-like γ^* . It would be interesting to have a HERA analysis similar to that of DØ to see if the energy dependence of the gap fraction is weaker, closer to x-independent, than at Fermilab. With respect to the DØ data the energy dependence of the gap fraction may be reflecting exactly the same phenomenon as observed in the energy dependence of the single diffraction cross section discussed in Sect. 2.2. While the inclusive two-jet cross section increases at higher energies, because of the growth in the parton densities, the energy dependence of the gap cross section is likely to be much weaker because of the increasing blackness of central proton-antiproton collisions as already seen in the single diffractive cross section.

5 BFKL searches

The hard (BFKL) pomeron or, equivalently, BFKL evolution shows up simply only in single transverse momentum hard scale processes. Thus in hadron-hadron collisions or in deep inelastic scattering where a soft scale, the size of the hadron (proton), is present a special class of events must be taken in order to isolate BFKL dynamics. Since this is generally very difficult to do experimentally it is perhaps useful to remind the reader why BFKL dynamics is so interesting for QCD and why it is worth the considerable effort necessary to uncover it.

There are at least two important reasons why hard single scale high energy scattering is interesting. (i) It is a high energy scattering problem that may be soluble, or nearly soluble. (ii) BFKL evolution leads to high parton densities and thus into a new domain of nonperturbative, but weak coupling, QCD. As parton distributions evolve from a momentum fraction x_1 to a smaller momentum fraction x_2 , all at a fixed transverse momentum scale, BFKL dynamics gives the rate of increase of those (mainly gluon) densities. This evolution is illustrated in Fig. 7. When gluon densities reach a density such that on the order of $1/\alpha$ gluons overlap, perturbation theory breaks down and one enters a new regime of strong field, $F_{\mu\nu} \sim 1/g$, QCD. While it is unlikely that one can reach such densities at Fermilab or HERA at truly hard transverse momentum scales one should at least be able to see the approach to these high densities through BFKL evolution.

Inclusive two-jet cross sections at Fermilab and forward single jet inclusive cross sections at HERA can be

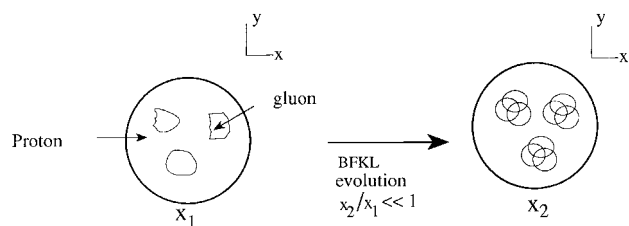


Fig. 7. A schematic picture of BFKL evolution

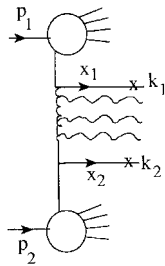


Fig. 8. Two-jet inclusive production

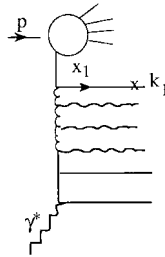


Fig. 9. Single forward jet production at HERA

used to measure the BFKL intercept [39-43]. These processes are illustrated in Figs.8 and 9 respectively where k_1 and k_2 represent measured jets. In proton-antiproton collisions one chooses $k_{1\perp}, k_{2\perp} > M$, a fixed hard scale, while in deep inelastic scattering $k_{1\perp}$ is chosen to be on the order of Q , the photon virtuality. For the hadron-hadron case

$$\sigma_{2-jet} = f(x_1, x_2, M^2) \frac{e^{(\alpha_P-1)\Delta Y}}{\sqrt{\Delta Y}} \quad (5.1)$$

while for deep inelastic scattering

$$\sigma_{jet} = f(x_1, Q^2) \frac{e^{(\alpha_P-1)\ln x_1/x}}{\sqrt{\ln x_1/x}} \quad (5.2)$$

with x_1 and x_2 being the longitudinal momentum fractions of the measured jets. $\alpha_P - 1 = \frac{4\alpha N_c}{\pi} \ln 2$ and the f 's in (5.1) and (5.2) are known in terms of the quark and gluon distributions of the proton and antiproton. In (5.1) ΔY is the rapidity difference between the two measured jets. One can get a measurement of $\alpha_P - 1$ in (5.1) by varying ΔY with x_1, x_2 and M^2 fixed, and this can be done at Fermilab by comparing the inclusive two-jet cross section at different incident energies. In (5.2) one can measure $\alpha_P - 1$ by varying x for fixed x_1 and Q^2 .

Sometime ago H1 [11,12] presented an analysis showing σ_{jet} increasing by about a factor of four as x goes from about 3×10^{-3} to about 7×10^{-4} for $k_{1\perp} > 3.5 GeV$. This is a growth quite a bit faster than given in conventional Monte Carlos and much faster than the growth from single gluon exchange between the measured jet and the quark-antiquark pair coming from the virtual photon. The growth is comparable to that given in (5.2), for $\alpha_P - 1 \approx 1/2$, however, the comparison is not completely convincing because a comparison of partonic energy dependences, from (5.2), with hadron final states is not very reliable when $k_{1\perp}$ is as small as in the H1 analyses.

Recently ZEUS [13] has completed an analysis of this process. Since the ARIADNE Monte Carlo gives a good fit to the ZEUS data this Monte Carlo is used to unfold the hadronization and thus get a better comparison with BFKL evolution. The data agree much better with BFKL evolution than with the Born term or with next-to-leading order QCD calculations. A definitive comparison with BFKL dynamics is hindered by the lack of ability to include hadronization corrections along with the BFKL evolution. One can hope that the situation will soon improve in this regard.

A new $D\bar{O}$ analysis [9] comparing 1800 GeV and 630 GeV data for $k_{1\perp}, k_{2\perp} \geq 20 GeV$ gives $\alpha_P = 1.35 \pm 0.04$ (*stat*) ± 0.22 (*syst*) when (5.1) is used to fit the data. The strength of the $D\bar{O}$ analysis is that $k_{\perp} > 20 GeV$ which makes uncertainties due to jet definition minimal. Weaknesses of the analysis are the large systematic error and the smallness of ΔY , equal to 2, at the lower energy. We can hope that the systematic errors will come down in the near future.

Overall, I think the BFKL searches are encouraging but not yet definitive. The fact that all three analyses suggest a strong increase with energy of reliable quantities for isolating BFKL effects is certainly positive. An attempt will also be made to measure $\alpha_P - 1$ at LEP [44] in the next year by measuring the $\gamma^* - \gamma^*$ total cross section. This is a very clean process, although the cross section is rather small.

6 Higher order corrections to BFKL evolution

In general in QCD next-to-leading corrections are very important. It is only after next-to-leading corrections have been calculated that scales have a real meaning and normalizations can be trusted. In the case of BFKL evolution the next-to-leading corrections are also important to show that, in principle, corrections to the BFKL answer can be calculated, thus making single scale high energy scattering systematically calculable in QCD.

There has been a long program [14, 45-47], led by the work of V. Fadin and L. Lipatov, to calculate the next-to-leading corrections to BFKL evolution and it now appears that program may be coming to completion. When the work is finished one should get the next correction to α_P as well as next-to-leading resummations for anomalous dimension and coefficient functions. If one writes

$$\alpha_P = \frac{4N_c}{\pi} \ln 2 \alpha(Q) [1 - c\alpha(Q)] \quad (6.1)$$

then there is the suggestion c may be near 3, a very large correction, although there is some work yet to be done before one can accept this number with confidence [14].

For the anomalous dimension matrix one writes

$$\gamma_n = \sum_{i=1}^{\infty} \gamma_{ni}^{(0)} \left[\frac{\alpha N_c}{\pi(n-1)} \right]^i + \alpha \sum_{i=1}^{\infty} \gamma_{ni}^{(1)} \left[\frac{\alpha N_c}{\pi(n-1)} \right]^i + \dots \quad (6.2)$$

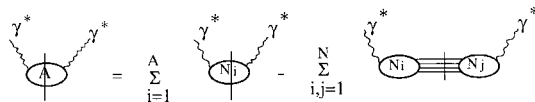


Fig. 10. Deep inelastic scattering on a nucleus in terms of single and double s terms

where the first series represents the leading order (BFKL) answer. We should soon know the second series, the constants $\gamma_{ni}^{(1)}$ along with similar terms for the coefficient functions. When the BFKL corrections are known at next-to-leading order we should reap several benefits. (i) A better understanding of the importance of BFKL (resummation) effects in νW_2 should be possible. Recall, that as a two-scale process BFKL dynamics does not directly govern the small- x behavior of νW_2 . However, BFKL effects are certainly present and can be systematically included through resummations such as the one indicated in (6.2). When the next-to-leading corrections are known we should begin to get a reliable indication of the importance of these resummation effects in the HERA regime. (ii) The next-to-leading resummations should help us to better understand where the operator expansion is valid in small- x physics, that is at what x and Q^2 are coefficient and anomalous dimension functions sufficiently safe from diffusion effects to be reliably calculated perturbatively [47, 48].

7 Nuclear reactions

7.1 Nuclear shadowing in deep inelastic scattering

Nuclear shadowing in deep inelastic scattering is known to be a leading twist phenomenon [29] and thus dominated by soft physics, at least at current x -values. In the DGLAP formalism shadowing effects are put into initial parton distributions. However, when shadowing is not too strong it can be calculated from diffractive deep inelastic scattering using the Gribov-Glauber formalism as illustrated in Fig. 10. In that figure the left-hand part represents the imaginary part of the forward γ^* -scattering amplitude which, by the optical theorem, is equal to the γ^* - A total cross section or, equivalently, νW_2 . The first term on the right-hand side of Fig. 10 represents the incoherent scattering off the A nucleons in the nucleus, the nucleons being labeled by N_i . The second term on the right-hand side of the figure represents the double scattering term which is dominated by diffractive scattering, off nucleon N_i in the amplitude and N_j in the complex conjugate amplitude. So long as shadowing correction are not too large it should not be necessary to go beyond the double scattering term. Indeed, the double scattering term can be obtained from diffractive data at HERA while triple and higher scattering terms would involve the scattering of partonic systems in the nucleus, terms which cannot be reliably determined. Using a parametrization which fits the HERA diffractive data, a pretty good description of fixed target deep inelastic scattering off nuclei is obtained with the double scattering term giving a shadowing correction of about the right size [15].

7.2 J/ψ production in proton-nucleus and nucleus-nucleus collisions

Recently there has been much interest, and excitement, about the NA50 data [18]. In a nutshell one can summarize the experimental situation as follows: (i) All P-A and A-A collisions, except for Pb-Pb, *look like* J/ψ production in proton-proton collisions with absorptive final state interactions corresponding to a “ J/ψ ” cross section with nucleons of 7mb [16, 17]. (ii) Pb-Pb central collisions have a J/ψ cross section which is significantly suppressed with respect to (i) [18].

Khazzev and Satz [49] have suggested a picture that considers the system moving through the nuclei, after the hard collision which produces the $c\bar{c}$ pair, to be a $(c\bar{c}g)$ color singlet system which, if it suffers no reaction with the nuclear medium, turns into a J/ψ after the $(c\bar{c}g)$ system has passed through the material. The $(c\bar{c}g)$ system is supposed to have a size comparable to that of the J/ψ , but, because of the fact that it looks like an octet dipole formed from $(c\bar{c})_8$, and a gluon a cross section of 7mb is natural. This is an interesting picture, however, there are a lot of unanswered questions. (i) Where does the gluon in the $(c\bar{c}g)$ system come from? Does it come from the hard scattering or is it part of the gluon distribution of the incident hadron or nucleus? If it is the latter does this *enhance* J/ψ production in nuclear collisions because there are so many more spectator gluons at the impact parameter of the collision. (ii) How does the gluon know what size system to form with the $c\bar{c}$ since the $(c\bar{c}g)$ does not interact, due to a slowing down of the rate of interaction for high velocity states, while traversing the material? Why should the relevant size for the ψ and ψ' be the same? While a very interesting, and successful, phenomenology has developed around this picture it is important to determine whether the whole picture is reasonable from a QCD point of view.

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